

DEVELOPMENT OF TWO WHEELS SPACE-FRAME PLUG-IN HYBRID  
ELECTRIC MOTORCYCLE CHASSIS

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## **ABSTRACT**

This research deals with the development and analysis of the plug in hybrid electric motorcycle frame. The significance of this project is to offer additional space on the existing conventional motorcycle models. A 304 stainless steel hybrid electric motorcycle chassis was fabricated upon the completion of the Computer Aided Design (CAD) modelling as well as a Finite Element Analysis (FEA) specifically stress analysis of the modelled chassis. SolidWorks was used to conduct both the modelling and the stress analysis. The simulation results exhibited desirable minimum factor of safety which in turn ensures the structural integrity of the chassis. Round hollow 1.5 inch and 1 inch 304 stainless steel tubes were used to form the main part of the chassis. The tubes were rolled to conform specified design and joined by means of Tungsten Inert Gas (TIG) welding and Metal Inert Gas (MIG) welding.

## **ABSTRAK**

Kajian ini berkaitan dengan pembangunan dan analisis rangka motosikal hibrid elektrik. Kepentingan projek ini adalah untuk menyediakan ruang tambahan pada model motosikal konvensional yang sedia ada. Casis motosikal hibrid elektrik keluli tahan karat 304 dihasilkan apabila selesai model CAD serta analisis FEA khusus terhadap tekanan casis yang telah dimodelkan. SolidWorks telah digunakan untuk merekabentuk dan menganalisis tekanan. Keputusan simulasi menunjukkan faktor keselamatan melebihi tahap minimum dan memastikan integriti struktur casis. Tiub bersaiz 1.5 inci dan 1 inci jenis 304 tiub keluli tahan karat digunakan untuk bahagian utama casis. Tiub telah dibentuk dengan mematuhi reka bentuk yang telah ditetapkan dan disambungkan dengan cara kimpalan tungstens gas lengai (TIG) dan kimpalan logam gas lengai (MIG).

## TABLE OF CONTENTS

	<b>Page</b>
<b>EXAMINERS APPROVAL DOCUMENT</b>	ii
<b>SUPERVISOR’S AND CO-SUPERVISOR’S DECLARATION</b>	iii
<b>STUDENT’S DECLARATION</b>	iv
<b>ACKNOWLEDGEMENT</b>	vi
<b>ABSTRACT</b>	vii
<b>ABSTRAK</b>	viii
<b>TABLE OF CONTENTS</b>	ix
<b>LIST OF TABLES</b>	xi
<b>LIST OF FIGURES</b>	xii
<b>LIST OF SYMBOLS</b>	xiii
<b>LIST OF ABBREVIATIONS</b>	xiv
 <b>CHAPTER 1            INTRODUCTION</b>	
1.1                      Background Study	1
1.2                      Problem Statement	2
1.3                      Objectives	3
1.4                      Scopes	3
1.5                      Hypothesis	3
1.6                      Flow Chart	4
1.7                      Gantt Chart	4
 <b>CHAPTER 2            LITERATURE REVIEW</b>	
2.1                      Development of Plug-In Hybrid Electric Vehicle (HEV)	5
2.2                      Development of Plug-In Hybrid Electric Motorcycle (PHEM)	6
2.3                      Importance of Chassis	8
2.4                      Motorcycle Chassis Design	8

2.5	Material Selection	10
2.5.1	Stainless steel 304	10
2.6	Stress and Strain Analysis	11
2.6.1	Stress and Strain Analysis on Motorcycle	12
2.7	Failure Criteria	13
2.8	Governing Equation	14
2.8.1	Stress	14
2.8.2	Strain	15
2.8.3	Modulus of Elasticity	16
<b>CHAPTER 3</b>	<b>METHODOLOGY</b>	
3.1	Study and Conceptual Chassis Design	17
3.2	Computational Stress and Strain Analysis	20
3.3	Chassis Fabrication	22
<b>CHAPTER 4</b>	<b>RESULTS AND DISCUSSIONS</b>	
4.1	Chassis Design	24
4.2	Simulation Result	26
4.3	Chassis Fabrication	30
<b>CHAPTER 5</b>	<b>CONCLUSION AND RECOMMENDATION</b>	
5.1	Conclusion	35
5.2	Recommendation	36
<b>REFERENCES</b>		37
<b>APPENDICES</b>		39
A	Gantt Chart	39
B	Tools and equipments	40

**LIST OF TABLES**

<b>Table No.</b>	<b>Title</b>	<b>Page</b>
2.1	Symbols and parameters of motorcycle chassis geometry	9
2.2	Stainless steel 304 mechanical properties	11
3.1	Summary of failure criteria	21
4.1	Static load stress-strain analysis parameter	29

## LIST OF FIGURES

<b>Figure No.</b>	<b>Title</b>	<b>Page</b>
2.1	Main parameters of motorcycle chassis geometry	9
2.2	Load case distribution on the motorcycle frame	12
2.3	Stress diagram	14
2.4	Strain diagram	15
3.1	Modenas Jaguh 175cc	18
3.2	Isometric view of components assembly in CAD software	19
3.3	Metal Inert Gas (MIG) welding machine	40
3.4	Hand grinder machine	40
3.5	Angle grinder machine	41
3.6	Rolling machine	41
3.7	Tungsten Inert Gas (TIG) welding	42
4.1	Side view of chassis 3D design	26
4.2	Front view of chassis 3D design	27
4.3	Isometric view of chassis 3D design	27
4.4	Chassis 3D model with fixtures and loads	28
4.5	Chassis 3D model meshing	29
4.6	Stress simulation result (von Mises Stress)	30
4.7	Strain simulation result	30
4.8	Displacement simulation result	31
4.9	Factor of safety simulation result	32
4.10	Jig frame with assembling front and rear wheel	33
4.11	Hollow stainless steel that had been rolled	33
4.12	Frontal part joining	34
4.13	Middle part joining with multipurpose jig	34
4.14	Rear part joining side view	35
4.15	Rear part joining isometric view	35
4.16	Complete chassis design without support	36

## LIST OF SYMBOLS

### Greek symbols

$\varepsilon$	Strain [unitless or %]
$\sigma$	Stress [Pa]
$\sigma_{\text{von Mises}}$	Von Mises Stress [Pa]
$\sigma_{\text{limit}}$	Yield strength [Pa]

### Symbols

$\Delta L$	Elongation of length [m]
F	Force [N]
L	Length [m]
E	Modulus of elasticity [Mpa]
A	Surface area [m <sup>2</sup> ]



## LIST OF ABBREVIATIONS

BEVs	Battery Electric Vehicle
CO <sub>2</sub>	Carbon Dioxide
CO	Carbon Monoxide
CAD	Computer Aided Design
CAE	Computer Aided Engineering
CVs	Conversional Vehicle
FOS	Factor of Safety
FEA	Finite Element Analysis
HEV	Hybrid Electric Vehicle
HC	Hydrocarbon
MIG	Metal Inert Gas
PHEM	Plug-in Hybrid Electric Motorcycle
PHEV	Plug-in Hybrid Electric Vehicle
TIG	Tungsten Inert Gas

## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 BACKGROUND STUDY**

A hybrid electric vehicle (HEV) consist of two or more power source (Gao Y et al., 2005) namely, internal combustion engine and an electric motor in order to improve its fuel efficiency (Huang KD and Tzeng S-C, 2004) and the reduction of harmful emissions (Doucette RT and McCulloch MD, 2011). A plug-in hybrid vehicle (PHEVs) is an HEV with the ability to recharge its energy storage system with the supply of electricity from the electric utility grid (Tony Markel, 2006). The terminology between PHEV and HEV can be classified further into charge-sustaining mode, charge-deleting mode, all-electric range (AER), electrified Miles, PHEV<sub>xx</sub>, SOC, degree of hybridization and utility factor (Tony Markel, 2006).

The hybrid electric motorcycle are introduced as motorcycles are the major mode of transportation especially in South Asia and Asia region (Yuan-Yong Hsu, 2009). Motorcycles are favoured due to limited space, short daily trip distances, population density, easy operation and maintenance (Shaik Amjad, 2010). The number of motorcycles has increased by 0.35 million per year for domestic sales and 1 million for export into South Asia market (Chia-Chang Tong, 2007). The development of hybrid electric motorcycles are driven by the 'go green' technological push, economic sense as well as to reduce harmful exhaust emissions (Yuan-Yong Hsu, 2009).

In the design of plug-in hybrid electric motorcycle (PHEM), the chassis plays a significant role as it supports the powertrain components, drivetrain parts and rider. A chassis is essentially the skeleton of a motorcycle. It must be straight to provide a secure

mounting for the steering apart from proper wheel alignment. The frame must be structurally sound to support the weight of the rider, the engine and the other components attached to it (Edward Abdo, 2009).

One of the chassis design types of the street motorcycles is the chopper or feet forward type. This type of chassis is characterized by the footrests being forward from the seat, long forks and low seat height. The handlebars may be higher as compared to the seat which is often positioned low. The riding position is as such that the legs of the rider are extended forward.

## **1.2 PROBLEM STATEMENT**

The integration of a plug-in hybrid electric motorcycle (PHEM) components in a limited space of a motorcycle frame is indeed a challenging task. Therefore, the design of the chassis is important to ensure enough space is provided to mount all the components well.

The chassis design should also provide enough strength to support the powertrain components, rider and other forms of weight contributors. Plug-in hybrid electric motorcycle (PHEM) in essence are heavier compared to conventional motorcycles. Therefore it is essential that the chassis could withstand the a fore mentioned contributing loads apart from providing adequate support.

Hence, the combination of the above mentioned aspects as well as other factors such as ergonomics, economics and aesthetic sense are commendable in the design of such chassis.

### **1.3 OBJECTIVES**

The objectives for this project are as follows

- a. To develop a stainless steel hybrid electric motorcycle chassis
- b. To analyze the stress and strain distribution of proposed chassis design

### **1.4 SCOPES**

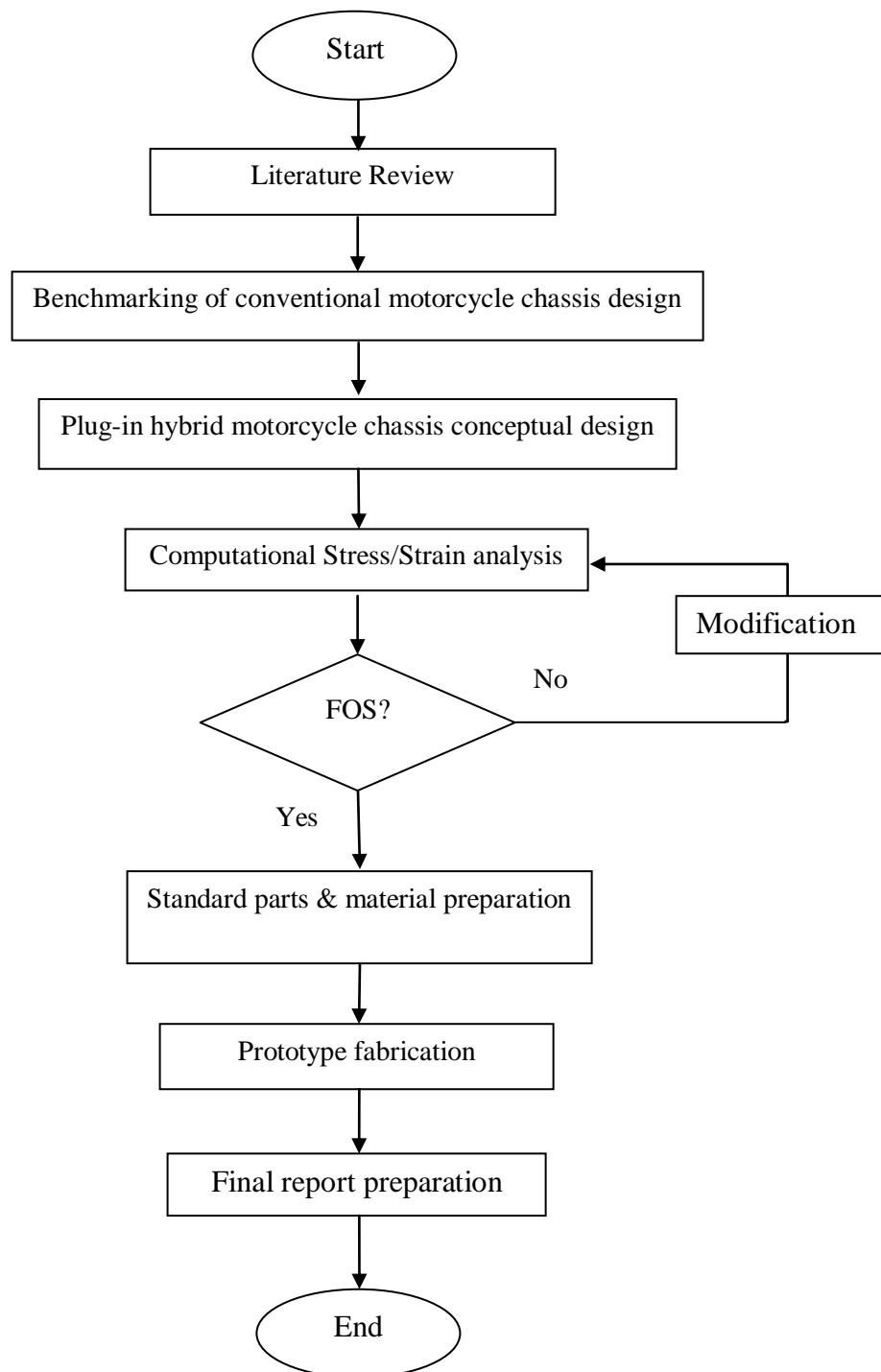
The scopes for this project is as follows

- a. Benchmark study on the conventional motorcycle chassis designs.
- b. Plug-in hybrid electric motorcycle chassis conceptual design.
- c. Computation of the stress and strain analysis by means of FEA.
- d. Standard component preparation.
- e. Chassis fabrication.

### **1.5 HYPOTHESIS**

Plug-in hybrid electric motorcycle chassis fabricated could fulfill the design considerations specified and a working prototype could be built.

## 1.6 FLOW CHART



## 1.7 GANTT CHART

Refer to APPENDIX A.

## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 DEVELOPMENT OF PLUG-IN HYBRID ELECTRIC VEHICLE (PHEV)**

Plug-in hybrid electric vehicles (PHEVs) have been proposed as a next step in the evolution of transportation technologies towards increased energy efficiency and less pollution (Romm and Frank, 2006; Suppes, 2006). They are similar to current hybrid gasoline-electric vehicles but have larger batteries and if their owners choose, charge their batteries from the electric grid and operate for some number of miles in all-electric mode. Ordinary hybrid vehicles have proven popular as sales in the U.S. have grown by over 80% annually since 2000, despite questions about the value of their fuel savings relative to the additional costs of the vehicles (see [www.hybridcars.com](http://www.hybridcars.com) and Lave and MacLean, 2002). Several companies now offer to convert ordinary hybrid vehicles into PHEVs and plan to sell retrofit kits, while at least one PHEV is being evaluated for sale in Europe

In a sense, PHEVs have two fuel tanks: they may use gasoline like a hybrid electric vehicle, or they may charge their batteries from the electric grid and run in all-electric mode until low battery charge leads the vehicle to switch to the gasoline-fueled hybrid electric mode. PHEVs promise to link the separate gasoline and electricity markets through the repeated marginal decisions of automotive fuel choice. As a result, PHEV owners should be more responsive than BEV owners to gasoline and electricity price signals, and unlike BEVs, the loads PHEVs place on the electric power system are discretionary because a PHEV can always operate on gasoline. In addition, because PHEVs are much more similar to conventional gasoline vehicles (CVs) than to BEVs in terms of technology and consumer experience, and are likely to have a smaller cost

premium than BEVs, consumers may adopt PHEVs more readily. However, the flexibility PHEVs display in operation makes their implications for energy markets less straightforward to estimate.

According to the IEEE-USA Energy Policy Committee classification, a plug-in hybrid electric vehicle (PHEV) has at least a 4 kWh battery energy storage system; could run a 16.1 km length in electric-drive mode only; and has means to recharge the energy storage system from an external electricity source (see [www.ieeeusa.org/policy](http://www.ieeeusa.org/policy)). Electric cars are characterized by the use of one or more electric motors connected to a mechanical shaft or directly to the wheels. They have an energy storage system, but still have low autonomy. The hybrid ones also feature an internal combustion engine, smaller, which can provide power to the electric motor, pull the vehicle, or only work at higher speeds (K. Clement-Nyns et al., 2007). There are many PHEVs with different configurations.

The main differences refer to motor/engine coupling to powertrain, which could be a series or parallel hybrid (A. Emadi et al., 2008) and battery energy storage system models. Regarding the electric motors, it's more common to use synchronous units with permanent magnets and rated power varying from 50 kW to 160 kW. As electric motor development continues, some of the key areas are identified, such as multiple motor design concepts including variable-voltage traction motors and sintered or bonded magnets for permanent magnet motors. Besides increasing performance, research efforts are being made to reduce materials, parts, and manufacturing costs (G. Wirasingha et al., 2008).

## **2.2 DEVELOPMENT IN PLUG-IN HYBRID ELECTRIC MOTORCYCLE (PHEM)**

In many urban areas of Asia especially India, two wheelers are quite popular mode of transportation because of limited space, short daily trip distance, easy in operation and maintenances (Sheu Kuen-Bao and Hsu Tsung-Hua, 2005). Statistics show that the number of two wheelers has multiplied by a factor of three times during this period and account for nearly two-third of the total vehicle population (Sheu Kuen-

Bao and Hsu Tsung-Hua, 2006). The pollutants, such as carbon monoxide (CO) and hydrocarbons (HC), produced by motorcycles account for approximately 10% of the total annual amount of pollution emissions in Taiwan (Yuan-Yong Hsu, 2010). In order to reduce emissions and go green technology of two wheelers, development in hybrid technology in two wheelers has been introduced (Yuan-Yong Hsu, 2010).

The Environmental Protection Administration of the ROC has implemented some policies to reduce air pollution, such as the strict exhaust standards for gasoline vehicles, an electric motorcycle development action plan, and a subsidy for purchasing electric scooters (Sheu Kuen-Bao and Hsu Tsung-Hua, 2005). To facilitate this, the government and industry have been applying fuel-cell technology to power scooters (Wang JH et al., 2000). However, the goal of replacing polluting combustion-engine motorcycles with battery powered ones has not been successful in Taiwan (Tso C and Chang S-Y, 2003).

Another approach to reduce both pollution and get better performance is to utilize a hybrid concept of internal-combustion engine and battery at this stage. Over the past few years, hybrid electric vehicles (HEVs), primarily automobiles, have been actively developed and marketed (Brown LT et al., 2001). This study considers the design of a hybrid power-transmitting system that is suitable for motorcycles. In 1997, Honda Motors released a hybrid two-wheeler concept in the Tokyo motor show with the key goals of a 60% reduction in carbon dioxide (CO<sub>2</sub>) emission and 2.5 times better fuel-efficiency. In this system, a water-cooled 49 cc gasoline engine is packed with a DC brushless electric-motor together driving the rear wheel. The gasoline engine delivers power for high-speed performance and for hill climbing while the electric motor engages for low-speed cruising.

In 1999, AVL Company proposed a hybrid system that used a 50 cc carbureted lean-burn two-stroke engine with a 0.75 kW electric motor mounted on the engine crankshaft mainly to provide increased torque during acceleration (Sheu Kuen-Bao and Hsu Tsung-Hua, 2005). Matsuto and Wachigai also proposed a motorcycle hybrid-drive system (Matsuo T et al., 2000). The main components of this system consists of the two power sources of an engine and an electric motor, a traction drive continuously-variable



transmission (CVT), a final reduction drive and three clutches. The transmission shaft and the electric motor shaft are coaxial in series in the longitudinal direction of the vehicular body and in parallel with the crank shaft of the engine.

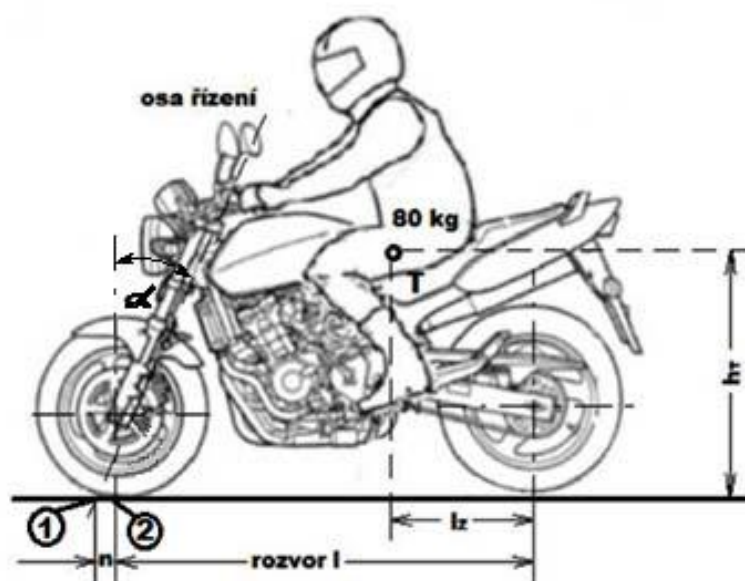
### **2.3 IMPORTANCE OF CHASSIS IN VEHICLE**

Basically chassis is considered as a framework to support the body, engine and other parts which make up the vehicle. Chassis lends the whole vehicle support and rigidity. Chassis usually includes a pair of longitudinally extending channels and multiple transverse cross members that intersect the channels. The transverse members have a reduced cross section in order to allow for a longitudinally extending storage space. The chassis has to contain the various components required for the motorcycle as well as being based around a rider position. Chassis play the important role in hybrid electric motorcycle to support weights. The hybrid electric motorcycle has an extra size in chassis compared with standard motorcycle design.

### **2.4 MOTORCYCLE CHASSIS DESIGN**

During development of the first motored bicycles, e.g. a steam powered Velociped Michaux-Perraux, emphasis was put on neither comfort nor driving stability. Only at the beginning of the 20th century, when the internal combustion engine has been patented by Dr. Otto, his assistant G. Daimler commenced to inquire into issues of driving stability and handling of a single-track vehicle in particular (Jakub Smiraus and Michal Richtar, 2011). Since then, the motorbike has been inexorably developed into more and more perfect machine with driving dynamics surpassing that of the majority of contemporary cars, which resulted in the occurrence of periods when engine power significantly outperformed capabilities of the chassis.

In chassis design, the most important thing that must be constraint is about the reference geometry on that chassis. The importance of reference chassis geometry is to produce new chassis in fully characteristic in design such as ergonomically, stability, maneuverability and safety consideration. Figure 2.1 shows the main parameter of reference motorcycle chassis geometry.



**Figure 2.1:** Main parameters of motorcycle chassis geometry

Source: Jakub Smiraus and Michal Richtar (2011)

**Table 2.1:** Symbols and parameters of motorcycle chassis geometry

Symbols	Parameters
$\alpha$	Rake angle
1	Point of wheel contact
2	Steering axis and ground intersection
$n$	Trail
$\ell$	Wheel-base
$h_T, l_z$	Center of gravity location

Source: Jakub Smiraus and Michal Richtar (2011)

The rake angle of the front fork indicates the angle between the steering axis and the ground plane. A smaller rake angle of the front fork results in a greater stabilizing effect on the front fork. The rake angle (angle of steering axis) lies within about  $24^\circ$  to  $30^\circ$  to the ground. Steering axis and ground intersection is the point of contact with the ground is indicated as wheel axis intersection perpendicular to the base of a stationary

bike at a point of their intersection. Trail is the distance between the steering axis and ground intersection and the point of wheel contact. The trail has a significant impact on the stability and handling of a motorcycle. The wheelbase is the distance between the rotation axis of the wheels in a straight-line drive. Center of gravity is determined by vertical and horizontal position.

The driving characteristics can be significantly affected by modification of the basic parameters. Therefore, if to customize the driving stability of an already constructed motorcycle, it must have the following options. Position can be adjusted the of centre of gravity. This can be achieved by change of a riding posture, which is, however, limited by placement of control and support elements of the motorcycle. Nevertheless, these changes are not always easy to control properly because it is impossible to encompass all the variety of passengers' weights. Further parameters of chassis are: wheel base, trail, rake or steering axis angle.

These dimensions are set, but if they were adjusted according to the driving conditions the overall driving stability as well as the steering response to the initiative of the driver could be influenced. Lengthening of both wheelbase and trail would result in an increased straight line stability, which would help to improve the comfort of long distance driving at higher speed. The opposite is assumed to be true as well. Shortening the latter mentioned parameters would lead to a decreased stability, yet at the same time a better handling, which is advantageous in the conditions of e.g. urban traffic. Such is the theory of these features impact on the motorcycle handling and driving characteristics.

## **2.5 MATERIAL SELECTION**

### **2.5.1 Stainless steel 304**

Stainless steel 304 is the ferrous metal and heat resistance categories. It comes from Austenitic Cr-Ni stainless steel. The characteristic of stainless steel 304 is a better corrosion resistance than Type 302. It also has a high ductility, excellent drawing, forming, and spinning properties. Essentially non-magnetic, becomes slightly magnetic

when cold worked. It is having a low carbon content means less carbide precipitation in the heat-affected zone during welding and a lower susceptibility to intergranular corrosion. Table 2.2 shows the mechanical properties of stainless steel 304.

**Table 2.2:** Stainless steel 304 mechanical properties

<b>Mechanical Properties</b>	<b>Metric</b>	<b>Comments</b>
Hardness, Brinell	123	Converted from Rockwell B hardness
Hardness, Knoop	138	Converted from Rockwell B hardness
Hardness, Rockwell B	70	Converted from Rockwell B hardness.
Tensile Strength, Ultimate	505 MPa	
Tensile Strength, Yield	215 MPa @Strain 0.200 %	
Elongation at Break	70 %	
Modulus of Elasticity	193 - 200 GPa	
Poissons Ratio	0.29	
Shear Modulus	86.0 GPa	
Charpy Impact	325 J	

Source: [www.matweb.com](http://www.matweb.com) (2013)

## 2.6 STRESS AND STRAIN ANALYSIS

Stress analysis is important in fatigue study and parts' life prediction where it aims to determine the critical point which has the highest stress (A. Rahman, 2008). Safety factor is used to provide a design margin over the theoretical design capacity. This allows consolidation of uncertainties in the design process. It is recommended by the conditions over which the designer has no control on the sources that are accounted for the uncertainties involved in the design process. In this study, safety factor rate with respect to design loading that is applied on the chassis has been investigated.

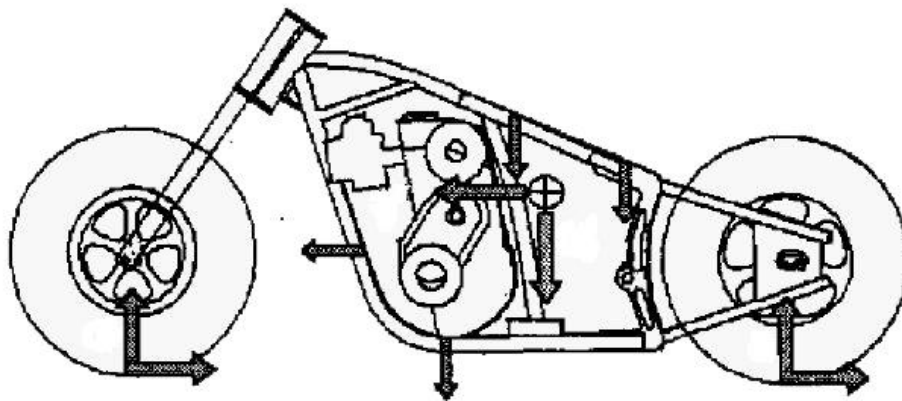
According to the journal of Design and Analysis on Eco Challenge Car Chassis by Mohd Hanif Mat (Mohd Hanif Mat, 2012), the chassis was modeled as beam elements with square and circular hollow sections using ABAQUS software. The chassis was simulated by using five different type of loading conditions. There are static

load (dead load) of vehicle supported at its wheel base, 4g load on the main hoop, 1.5g acceleration, 1.5g deceleration (braking) and 2500 Nm/degree of torsional loading.

### 2.6.1 Stress And Strain Analysis on Motorcycle

An analysis of the frame's components and their interactions is needed to obtain a lighter frame and improve its mechanical behaviour. The first functional prototype of the motorcycle frame was used using a traditional design approach, that is, to use manual intervention to fine tune the design parameters and run the simulation iteratively (Caldenjn. B. Diseiio, 2004).

However, this analysis can be done programmatically, simulating the performance of the frame using a FEA based software as a “black box” coupled with an optimization algorithm. In this study, they compared the latter approach with the traditional one based on manual “optimization” cycles. The FEA simulation models the mechanical behavior of the frame under an extreme load case to evaluate its performance according to the previously mentioned criteria.. Figure 2.2 shows the load case distribution on the motorcycle frame.



**Figure 2.2:** Load case distribution on the motorcycle frame

Source: Jorge E. Rodriguez et al., 2005

## 2.7 FAILURE CRITERIA

Failure of engineering materials can be broadly classified into ductile and brittle failure. Most metals are ductile and fail due to yielding. Hence, the yield strength characterizes their failure. Ceramics and some polymers are brittle and rupture or fracture when the stress exceeds certain maximum value. Their stress–strain behavior is linear up to the point of failure and they fail abruptly.

The stress required to break the atomic bond and separate the atoms is called the theoretical strength of the material. It can be shown that the theoretical strength is approximately equal to  $E/3$  where,  $E$  is Young's modulus (T.L. Anderson, 2006). However, most materials fail at a stress about one–hundredth or even one–thousandth of the theoretical strength. For example, the theoretical strength of aluminum is about 22 GPa. However, the yield strength of aluminum is in the order of 100 MPa, which is 1/220th of the theoretical strength.

In ductile material yielding occurs not due to separation of atoms but due to sliding of atoms (movement of dislocations). Thus, the stress or energy required for yielding is much less than that required for separating the atomic planes. Hence, in a ductile material the maximum shear stress causes yielding of the material.

In brittle materials, the failure or rupture still occurs due to separation of atomic planes. However, the high value of stress required is provided locally by stress concentration caused by small pre-existing cracks or flaws in the material. The stress concentration factors can be in the order of 100 to 1,000. That is, the applied stress is amplified by enormous amount due to the presence of cracks and it is sufficient to separate the atoms. When this process becomes unstable, the material separates over a large area causing brittle failure of the material.

Although research is underway not only to explain but also quantify the strength of materials in terms of its atomic structure and properties, it is still not practical to design machines and structures based on such atomistic models. Hence, we resort to phenomenological failure theories, which are based on observations and testing over a

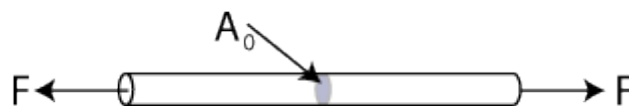
period of time. The purpose of failure theories is to extend the strength values obtained from uniaxial tests to multi-axial states of stress that exists in practical structures. It is not practical to test a material under all possible combinations of stress states. In the following, we describe some well-established phenomenological failure theories for both ductile and brittle materials.

## 2.8 GOVERNING EQUATION

### 2.8.1 Stress

The term stress is used to express the loading in terms of force applied to a certain cross-sectional area of an object. From the perspective of loading, stress is the applied force or system of forces that tends to deform a body. From the perspective of what is happening within a material, stress is the internal distribution of forces within a body that balance and react to the loads applied to it. The stress distribution may or may not be uniform, depending on the nature of the loading condition. For example, a bar loaded in pure tension will essentially have a uniform tensile stress distribution. However, a bar loaded in bending will have a stress distribution that changes with distance perpendicular to the normal axis.

Simplifying assumptions are often used to represent stress as a vector quantity for many engineering calculations and for material property determination. The word "*vector*" typically refers to a quantity that has a "magnitude" and a "direction". For example, the stress in an axially loaded bar is simply equal to the applied force divided by the bar's cross-sectional area.



**Figure 2.3:** Stress diagram

Source: <http://www.ndt-ed.org> (2013)

$$\sigma = \frac{F}{A} \quad (2.1)$$

Where,  $\sigma$  is stress [Pa]

F force [N]

A surface area [ $\text{m}^2$ ]

### 2.8.2 Strain

Strain is the response of a system to an applied stress. When a material is loaded with a force, it produces a stress, which then causes a material to deform. Engineering strain is defined as the amount of deformation in the direction of the applied force divided by the initial length of the material. This results in a unit less number, although it is often left in the unsimplified form, such as inches per inch or meters per meter. For example, the strain in a bar that is being stretched in tension is the amount of elongation or change in length divided by its original length. As in the case of stress, the strain distribution may or may not be uniform in a complex structural element, depending on the nature of the loading condition.



**Figure 2.4:** Strain diagram

Source: <http://www.ndt-ed.org> (2013)

$$\varepsilon = \frac{\Delta L}{L} \quad (2.2)$$